Stagnant Film Model for Effect of Diffusional Layer Thickness on Heat Transfer and Exerted Friction

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For large mass-transfer rates, heat and diffusional mass transfer, as well as exerted friction, are influenced by the extra flow due to the mass transfer (corresponding, effectively, to suction or injection). In the past, corrections for this influence have been derived from three approximate theories or models: the boundary layer model, the penetration model, and the film model. The film model is physically the most simple description of the effect of the additional induced velocity, traditionally referred to as "convective velocity," "bulk velocity" or "Stefan flow." The model idealizes the transition between the fluid properties and those at the wall as occurring entirely within a thin film next to the wall. In this film, the influence of convection parallel to the wall is furthermore neglected.

Though the correction factors supplied by the film model are based on a simple physical model, they are widely used in engineering applications. This is due to the fact that the physical accuracy of the correction factors is generally sufficiently high for engineering purposes, and they are easily applicable. Practical situations may involve flow in channels or along external surfaces in the presence of mass transfer [by suction/injection or (multicomponent) condensation/evaporation].

Stefan (1873) was the first to experimentally and theoretically study the diffusional mass transfer in a stagnant film, including the induced velocity. Gilliland and Sherwood (1934) and Colburn and Hougen (1934) applied the film model to forced convective diffusional mass transfer. In the latter article, sensible heat transfer was also considered, but the effect of the induced velocity on heat transfer was overlooked.

Ackermann (1937) examined the effect of the induced velocity on both heat and mass transfer in a film. Colburn and Drew (1937) did the same independently, and applied the film model to the case of flow in a closed channel. From this application, equations were derived for the change in bulk temperature and vapor concentration. Mickley et al. (1954) derived the three now well-known film model correction factors for mass, heat, and momentum transfer, which can be applied to any process of importance. Brouwers (1990) and Brouwers and Chesters (1992) reviewed the film model, and employed the model to predict both the exerted friction and the pressure drop of flow in a closed channel, via a local momentum balance.

The present stagnant film analysis revises the thermal (or Ackermann) correction factor for the effect of wall mass transfer on heat transfer. It is explained that for mixtures for which the diffusional film thickness is smaller than the thermal film thickness (or, equivalently, Sherwood number Sh_0 exceeds Nusselt number Nu_0), the conventional film theory is no longer applicable. Accordingly, the presented film model accounts for the actual part of the thermal film in which mass is transferred. This fundamental analysis of the stagnant film provides an alternative and novel thermal correction factor which is valid for all Sh_0/Nu_0 ratios.

Subsequently, on the basis of a similar consideration of the momentum equation of the film, an alternative frictional correction factor is put forward. This correction factor is applicable in cases where the frictional film thickness exceeds the thickness of the mass-transferring film.

Finally, the alternative thermal correction factor is used to predict fog formation of a mixture with $Sh_0/Nu_0 > 1$. The results are compared with earlier predictions based on the conventional film model and its pertaining correction factors.

Conventional Film Analysis

Consider a stagnant film through which there is a steady transfer of heat and mass in the y-direction. At the "wall" ($y = 0$), such as denoting the surface of a liquid film, the temperature is t_i and at a distance δ_t, the bulk temperature t_b is attained. Between the wall and $y = \delta_c$, mass is transferred. For the purposes of this analysis, the physical properties in the film are assumed to be constant and the mixture to behave as an ideal, incompressible, and Newtonian gas.

The energy equation of the film is

$$\rho c_p v \frac{dt}{dy} = k \frac{d^2 t}{dy^2}. \tag{1}$$

In Eq. 1 the velocity v is caused by mass transfer (such as by diffusion of vapor, hence, δ_c is henceforth referred to as diffusional film thickness) through the film.

Without mass transfer in the film ($v = 0$), Eq. 1 reduces to

$$\frac{d^2t}{dy^2} = 0. \tag{2}$$

Integrating this equation twice with respect to y yields:

$$t(Y) = C_1 Y + C_2. \tag{3}$$

In Eq. 3 the dimensionless coordinate Y ($0 \le Y \le 1$) has been introduced:

$$Y = \frac{y}{\delta_t}. \tag{4}$$

With application of boundary conditions:

$$t(Y = 0) = t_i, \tag{5}$$

$$t(Y = 1) = t_b, \tag{6}$$

the temperature in the film is obtained as:

$$t(Y) = Y(t_b - t_i) + t_i. \tag{7}$$

The heat transfer from the fluid to the wall is described by Fourier's law as:

$$q = k\frac{dt}{dy}\bigg|_{y=0} = \frac{k}{\delta_t}\frac{dt}{dY}\bigg|_{Y=0}. \tag{8}$$

In case of no mass transfer, the transferred heat through the film is simply given by:

$$q_0 = \frac{k}{\delta_t}(t_b - t_i), \tag{9}$$

which serves as reference level for the effect of the mass transfer on heat transfer.

If mass is transferred, two possible situations should be distinguished. Namely, the situation where the diffusional film thickness is larger than the thermal film thickness ($\delta_c > \delta_t$) and the case whereby $\delta_c < \delta_t$. In the first case Eq. 1 holds in the entire thermal film; this is the region $0 \le y \le \delta_t$. Hence Eq. 1 can be solved with application of Eq. 4:

$$t(Y) = \frac{C_4 e^{-\phi_t Y} - C_3}{-\phi_t}, \tag{10}$$

with as thermal dimensionless mass flux

$$\phi_t = -\frac{\delta_t \rho c_p v}{k}. \tag{11}$$

Application of boundary conditions 5 and 6 yields:

$$t(Y) = (t_b - t_i)\left(\frac{e^{-\phi_t Y} - 1}{e^{-\phi_t} - 1}\right) + t_i. \tag{12}$$

The heat transfer from fluid to wall is obtained by combining Eqs. 8, 9 and 12:

$$q = q_0 \frac{-\phi_t}{e^{-\phi_t} - 1}. \tag{13}$$

On a comparison of Eq. 13 with Eq. 9, the following film model correction factor is derived:

$$\Theta_t = \frac{-\phi_t}{e^{-\phi_t} - 1}. \tag{14}$$

This thermal correction factor is commonly referred to as Ackermann correction factor. It is the original result of Ackermann (1937) and has been widely used in engineering applications. To this end, the actual heat transfer (or Nusselt number Nu or Stanton number St) is obtained by multiplying the zero flux heat transfer (or Nu_0 or St_0) by the Ackermann correction Θ_t. In the way δ_t of Eq. 11 determined will be explained later.

In the discussion above, it has however been pointed out that this result is applicable only if $\delta_c \ge \delta_t$. In case $\delta_c < \delta_t$, an alternative analysis is required which reckons with the limited part of the thermal film in which mass is actually transferred. In the next section such an analysis is presented.

Alternative Film Analysis

In this section the energy equation in the film is solved for $\delta_c < \delta_t$. Subsequently, an alternative correction factor is derived which accounts for both the magnitude of the dimensionless mass flux ϕ_t and the actual mass-transferring film thickness.

To this end, the film is divided into a region I next to the wall in which mass transfer takes place ($0 \le y \le \delta_c$) and a region II in which the mass transfer is absent ($\delta_c \le y \le \delta_t$), as shown in Figure 1. If mass transfer is caused by diffusion between wall and bulk, then $y = \delta_c$ corresponds to the location where the vapor mass fraction attains the bulk mass fraction.

In region I Eq. 1 and boundary condition 5 hold. In region II Eq. 2 and boundary condition 6 are valid. Equations 1 and 2 can be integrated, the result being described by Eqs. 10 and 3, respectively. So Eq. 10 describes the temperature in region I and Eq. 3 in region II. Application of boundary conditions 5 and 6 yields:

$$C_4 = -\phi_t t_i + C_3, \tag{15}$$

$$C_2 = t_b - C_1. \tag{16}$$

In order to solve the energy equation in the entire region $0 \le Y \le 1$, two extra conditions are required. These conditions are provided by the gradient continuity condition of the temperature at $y = \delta_c$:

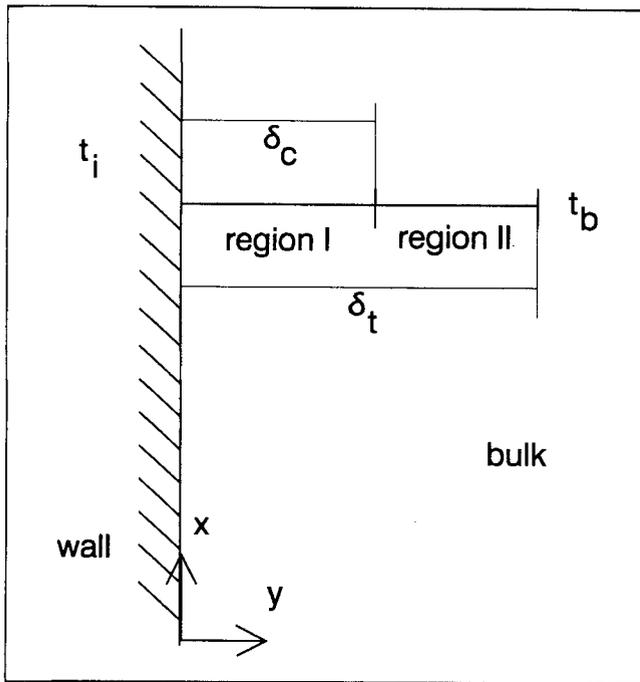$$\begin{array}{cc} t(Y = \gamma) = t(Y = \gamma) \\ \text{region I} \quad \text{region II} \end{array}, \tag{17}$$

Figure 1. Stagnant film.

$$\left.\frac{dt}{dY}\right|_{Y=\gamma} = \left.\frac{dt}{dY}\right|_{Y=\gamma}, \tag{18}$$

$$\text{region I} \quad \text{region II}$$

with

$$\gamma = \frac{\delta_c}{\delta_t}. \tag{19}$$

Application of Eqs. 17 and 18 to Eqs. 3 and 10 and substitution of Eqs. 15 and 16 yields:

$$C_1 = C_4 e^{-\phi_t \gamma}, \tag{20}$$

$$C_4 = \frac{-\phi_t(t_b - t_i)}{e^{-\phi_t \gamma}[1 - (1-\gamma)\phi_t] - 1}. \tag{21}$$

The heat flux to the wall follows from Eqs. 8, 1 and 21 as:

$$q = \frac{k}{\delta_t} C_4. \tag{22}$$

On a comparison of Eqs. 22 and 21 with Eq. 9, the following correction factor is established for $\delta_c < \delta_t$ (or $\gamma < 1$):

$$\Theta_t = \frac{-\phi_t}{e^{-\phi_t \gamma}[1 - (1-\gamma)\phi_t] - 1}. \tag{23}$$

In Figure 2 Θ_t is drawn against ϕ_t for $\gamma = 0$, 0.25, 0.5, 0.75 and 1. The figure reveals that for $\gamma < 1$, Θ_t is closer to unity than predicted by the conventional correction factor, which is

represented by Eq. 14. Both correction factors reduce to unity for ϕ_t tending to zero, that is, for vanishingly small mass-transfer rates.

Equation 23, regardless the value of ϕ_t also tends to unity for γ tending to zero, that is, when the effective mass-transferring layer is reduced to zero. This effect is not recognized by Eq. 14. Accordingly, Eq. 14 is recommended for $\delta_c/\delta_t \ge 1$ and Eq. 23, derived here for the first time for $\delta_c/\delta_t \le 1$. For $\delta_c = \delta_t$ (that is to say $\gamma = 1$) Eqs. 23 and 14 coincide, which would be expected.

To apply the film model, for the film thicknesses:

$$\delta_c = \frac{x}{Sh_0}, \tag{24}$$

$$\delta_t = \frac{x}{Nu_0}, \tag{25}$$

are substituted for flow along external surfaces, where x is a coordinate along the surface.

For flow in confined spaces on the other hand

$$\delta_c = \frac{D_h}{Sh_0}, \tag{26}$$

$$\delta_t = \frac{D_h}{Nu_0}, \tag{27}$$

are employed to apply the film model; D_h then represents the hydraulic diameter of the channel. For most flow situations, either laminar or turbulent, free and/or forced convective, the zero flux Nusselt (or Stanton) and Sherwood numbers are documented.

Equations 24–27 reveal that

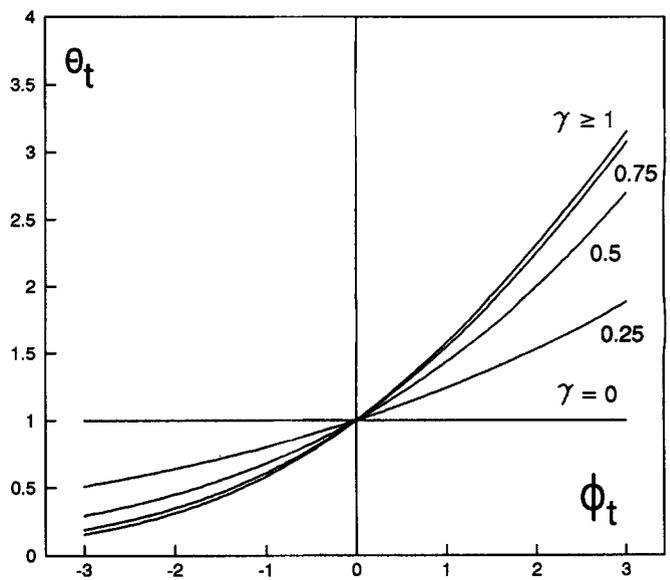$$\frac{Nu_0}{Sh_0} = \frac{\delta_c}{\delta_t} = \gamma. \tag{28}$$

Figure 2. Thermal correction factor Θ_t as a function of ϕ_t and for various γ.

So for $Nu_0/Sh_0 < 1$, the alternative correction factor is recommended. For $Nu_0/Sh_0 \geq 1$, the conventional Ackermann correction factor 14 is still prevailing and applicable. Equation 28 reveals that γ is governed by the zero flux Nusselt and Sherwood numbers and hence is a prescribed parameter.

Momentum Transfer

Mickley et al. (1954) were the first to derive and apply a film model correction factor for the effect of wall mass transfer on exerted friction. Brouwers (1990) and Brouwers and Chesters (1992) recapitulated their derivation and applied the model to the case of closed channel flow. The conventional friction factor is

$$\Theta_u = \frac{-\phi_u}{e^{-\phi_u} - 1}, \tag{29}$$

with as frictional dimensionless mass flux

$$\phi_u = -\frac{\delta_u \rho \upsilon}{\eta}. \tag{30}$$

The frictional correction 29 was based on the assumption that mass transfer takes place in the entire frictional film thickness δ_u, which is defined as:

$$\delta_u = \frac{2\eta}{\rho u_b f_0}, \tag{31}$$

with f_0 as zero flux Fanning friction factor. The actual friction factor (or friction) follows from multiplying the zero flux friction factor f_0 (or friction) by Θ_u. In the previous section, we have seen that for $\delta_c < \delta_u$ an alternative analysis is required.

A similar analysis for the momentum equation as carried out for the energy equation presented in the previous section yields as correction factor:

$$\Theta_u = \frac{-\phi_u}{e^{-\phi_u \chi}[1-(1-\chi)\phi_u] - 1}, \tag{32}$$

for:

$$\chi = \frac{\delta_c}{\delta_u} < 1. \tag{33}$$

Equation 32 is the same as Eq. 23, so the lines in Figure 2, with Θ_t replaced by Θ_u, ϕ_t by ϕ_u and γ by χ, adequately represents Eq. 32.

Summarizing, Eq. 29 is recommended for $\delta_c \geq \delta_u$, whereas Eq. 32 is recommended for $\delta_c \leq \delta_u$. In contrast to the classical correction factor 29, Eq. 32 reckons with the part of the frictional film in which mass transfer is actually effective. Likewise γ, χ depends on two zero flux properties, namely f_0 (Eq. 31) and Sh_0 (Eqs. 24 and 26), and is therefore also a prescribed parameter.

Fog Formation

In this section the alternative correction factor is employed to investigate fog formation in a mixture for which $Nu_0/Sh_0 < 1$. In case of condensation ($\phi_t > 0$) Figure 2 reveals that heat transfer is smaller for $\gamma < 1$ in comparison to the conventional film model (curve pertaining to $\gamma \geq 1$). In other words, the temperature gradient near the wall is flattened. The results of the alternative analysis should therefore also be used if fog formation in gas mixtures is investigated. A practical case of the determination of fog formation with the help of the tangency condition is discussed below.

Johnstone et al. (1950) introduced the tangency condition to determine fog (or mist) formation in binary mixtures in cooled channels. In a subsequent article this condition was improved by Brouwers (1991). The tangency condition is based on the heat and diffusional mass-transfer rates to a condenser wall, and the slope of the saturation line of the vapor component at wall temperature. The transfer rates follow from the energy and diffusion equation in a stagnant film next to the wall. With the aid of the improved tangency condition of Brouwers (1991), fog formation in mixtures of nitrogen and water vapor could be excellently explained. These wall condensation experiments were originally reported by Johnstone et al. (1950).

On the other hand, the experiments of Johnstone et al. (1950) with mixtures of nitrogen and n-butyl alcohol could not be explained satisfactorily. Fog was observed experimentally, but could not be explained theoretically with the tangency condition. The Lewis number of these mixtures, in contrast to the mixtures of nitrogen and water vapor, is larger than unity ($1.41 \leq Le \leq 1.77$). Furthermore, it was explained by Johnstone et al. (1950) and Brouwers (1991) that for said experiments concerning laminar flow in the entrance region of a tube holds:

$$\frac{\delta_c}{\delta_t} = \frac{Nu_0}{Sh_0} = Le^{-1/3}. \tag{34}$$

From Eqs. 19 and 34, it follows that $0.82 < \gamma < 0.90$ for said experiments. This implies that for these experiments the alternative thermal correction factor governed by Eq. 23 prevails. So, for the experiments with n-butyl alcohol Θ_t^+ in the tangency condition (Eq. 16 of Brouwers, 1991) should be determined with Eq. 23 instead of Eq. 14. The dimensionless mass flux in Eq. 23, see Brouwers (1991) for derivation and notation, then reads:

$$\phi_t = -\frac{c_{p,\upsilon}^+ \delta_t}{c_p^+ Le \delta_c} \ln\left(\frac{1-c_b^+}{1-c_a^+}\right). \tag{35}$$

As Θ_t^+ following the alternative film model is smaller than the conventional film model (which was used by Brouwers, 1991), the critical wall temperature at which fog is formed, denoted by t_a, will be higher and the observed fog formation probably explained. For some experiments with n-butyl alcohol, fog was namely observed while the theoretical t_a was still below the measured interface temperature, denoted by t_i.

Hence, t_a has been recomputed with Θ_t^+ following Eq. 23

and all other values unchanged. Indeed the newly determined t_a are higher, but a few 0.1°C only. These slightly increased t_a are still below t_i. Accordingly, the observed fog formation in the mixtures of nitrogen and n-butyl alcohol cannot be explained yet. Apparently, the presented fog formation problem is not suited to prove the alternative film model. This can be attributed to:

- The Le number of mixtures of nitrogen and n-butyl alcohol does not sufficiently deviate from unity
- The considered flow situation. For forced convective laminar flow in the entrance region of a tube the exponent of Le does not differ enough from zero (see Eq. 34, the exponent of Le is $-1/3$ only)
- The mass flux and related ϕ_t is not large enough (0.188 $\leq \phi_t \leq 0.468$). This can be attributed to the relatively small difference between vapor interface mole fraction ($c_i^+ \geq 0.010$) and vapor bulk mole fraction ($c_b^+ \leq 0.182$) during the experiments.

Accordingly, a better validation of the alternative film will be possible if γ is reduced and ϕ_t enlarged.

Finally, it must be stressed again, as $Le < 1$ for the mixtures of nitrogen and water vapor and hence $Nu_0/Sh_0 > 1$ the analysis and conclusions of Brouwers (1991) with regard to these mixtures remains unaltered.

Conclusions

The film model provides correction factors for the effect of wall mass transfer (such as by suction or injection) on heat transfer and exerted friction between fluid and wall. In the present analysis, it has been demonstrated that the conventional correction factors do not account for the thickness of the mass-transferring layer (or diffusional film thickness). Hence, the classical correction factors are not generally applicable for $Sh_0/Nu_0 > 1$ and an improved film analysis is required.

Here, new correction factors have been derived which account for both the magnitude of the mass-transfer rate and the thickness of the mass-transferring film. These correction factors represented by simple analytical expressions reveal the substantial influence of the actual diffusional film thickness on transport phenomena.

Furthermore, the improved thermal correction factor has been applied in the tangency condition. This condition is used to predict fog formation in mixtures of nitrogen and n-butyl alcohol. The experimental results of Johnstone et al. (1950) cannot be explained with the conventional nor alternative thermal correction factor. A better validation of the presented model will be possible with experiments where Sh_0/Nu_0 is less close to unity and/or the mass flux is larger.

Notation

c^+ = vapor mole fraction
c_p = specific heat, J·kg^{-1}·K^{-1}
c_p^+ = molar specific heat, J·mol^{-1}·K^{-1}
$C_1, ..., C_4$ = integration constants, K
$\mathfrak{D}$ = diffusion coefficient, m^2·s^{-1}
D_h = hydraulic diameter, m
k = thermal conductivity, W·m^{-1}·K^{-1}
Le = Lewis number, $k/\rho c_p \mathfrak{D}$
q = heat flux at wall, W·m^{-2}
t = temperature, K
u = component of velocity in the direction of x, ms^{-1}
v = component of velocity in the direction of y, ms^{-1}
x = coordinate, m
y = coordinate, m

Greek letters

γ = dimensionless film thickness, Eq. 19
δ = film thickness, m
η = dynamic viscosity, Pa·s
θ = correction factor
ρ = density, kg·m^{-3}
ϕ = dimensionless wall mass flux (Eqs. 11, 30 and 35)
χ = dimensionless film thickness (Eq. 33)

Subscripts

a = critical interface condition for fog formation
b = bulk
c = diffusional
i = interface
t = thermal
u = frictional
v = vapor
0 = pertaining to zero mass flux

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